Objectives:

- Find a function that models a problem and apply the techniques from 4.1, 4.2, and 4.3 the find the optimal or "best" value.

Suggested procedure:

Step 1. Draw a picture! Label variables and known quantities.

Step 2. Decide what quantity we want to maximize (or minimize).

Step 3. Find a formula for the quantity that we want to maximize (or minimize).

Step 4. Use constraints to turn our formula into an equation in <u>one</u> variable.

Step 5. Find the domain.

Step 6. Find the global minimum (or maximum).

(a) If we are looking in a closed interval: substitute endpoints and critical points into the function and choose the largest (or smallest) value.

(b) If we are looking in an open interval: Hope there is only one critical point, show there is a local maximum (or minimum) there, conclude it is also a global maximum (or minimum).

Step 7. Remember to answer the original question clearly and completely!

Example 1. Find two non-negative numbers whose sum is 200 and whose product is maximum.

Example 2. The corners are cut our of an $8\frac{1}{2}'' \times 11''$ piece of paper and it is folded into a box. What size squares should be removed to maximize the volume?

Example 3. A rectangle is inscribed in the triangle with vertices $(0,0)$, $(4,0)$, and $(0,8)$ with one side of the rectangle on lying on the x-axis and one side of the rectangle lying on the y-axis. What is the maximum area of the rectangle?

Example 4. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Example 5. A rectangular mural will have a total area of 24 ft^2 which includes a border of 1 ft on the left, right, and bottom and a border of 2 ft on the top. What dimensions maximize the total paintable area inside the borders.

Example 6. A can is made to hold 1 liter of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

Example 7. A glass fish tank is to be constructed to hold 72 ft^3 of water. The top is to be open. The width will be 5 ft but the length and the depth are variable. Building the tank costs \$10 per square foot for the base and \$5 per square foot for the sides. What is the cost of the least expensive tank?

Example 8. A baseball team plays in a stadium that holds $55,000$ spectators. with ticket prices at \$10, the average attendance has been $27,000$. Some financial experts estimated that prices should be determined by the function $p(x) = 19 - \frac{1}{3000}x$ where x is the number of tickets sold. What should the price per ticket be to maximize revenue?